3D Modeling of heterogeneous and anisotropic superconducting media

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In this paper, we investigate the 3-D computation of the current distribution in an heterogeneous superconducting media with a discontinuous anisotropic $E-J$ constitutive power law. The resulting 3-D vectorial non-linear diffusion problem satisfied by the electric field is solved in each region of the media with a symmetric interior penalty discontinuous Galerkin method combined with a 3D semi-implicit scheme recently proposed for the 3-D modeling of homogeneous bulk superconductors. A numerical example on a media consisting of two different anisotropic superconducting regions inside a conductive matrix is computed. While the critical electric field E_c is uniform, both critical current density J_c and power law exponent n are uniform in each region but discontinuous in the media. It can be seen that the numerical method proposed is stable and convergent.

***Index Terms*—Anisotropic constitutive power law, Discontinuous Galerkin Method, Heterogeneous superconducting media, Maxwell equations.**

I. INTRODUCTION

NUMEROUS superconductivity applications involve the use of coated superconductors [4]. They consist of superconducting films deposited on metal substrates and the coating process allows them to carry large electrical currents. Moreover, multifilamentary superconductors, whether twisted or not, in a resistive matrix have been developed for alternating applications [3].

Heterogeneity and anisotropy are therefore properties to consider in the scope of the 3-D modeling of superconductors. A numerical method based on the discontinuous galerkin method, proposed in [6], successfully modeled in 3-D homogeneous and isotropic bulk superconductors.

In this paper, we extend that method to the general case of heterogeneous and anisotropic superconducting media whose domain Ω consists of non-overlapping superconducting sub-domains $\bigcup\limits_{r=f} \Omega_r$ embedded in a non-overlapping resistive matrix sub-domain Ω_m with $\partial\Omega = \partial\Omega_m$. While the critical electric field E_c is uniform in the whole domain, each sub-domain Ω_r is characterized by a critical current density $J^i_{r,c}$ in each direction of the space ($i = 1, 2, 3$) and a power law exponent n_r ($n_m = 1$ since the matrix sub-domain Ω_m is a normal conductor).

II. THE DIFFERENTIAL SYSTEM

The electrical behavior of each sub-domain Ω_r of the heterogeneous and anisotropic superconducting media is defined by the following constitutive equations :

$$\overrightarrow{J} = \boldsymbol{\sigma_r}(E_c, J^i_{r,c}, n_r)\overrightarrow{E} \quad \text{or} \quad \overrightarrow{E} = \boldsymbol{\rho_r}(E_c, J^i_{r,c}, n_r)\overrightarrow{J} \tag{1}$$

with

$$\boldsymbol{\sigma_r}(E_c, J^i_{r,c}, n_r) = \begin{bmatrix} \sigma_r^{11} & 0 & 0 \\ 0 & \sigma_r^{22} & 0 \\ 0 & 0 & \sigma_r^{33} \end{bmatrix}, \sigma_r^{ii} = \frac{J^i_{r,c}}{E_c} \left\| \frac{\overrightarrow{E}}{E_c} \right\|^{1-\frac{1}{n_r}} \tag{2}$$

and

$$\boldsymbol{\rho_r}(E_c, J^i_{r,c}, n_r) = \begin{bmatrix} \rho_r^{11} & 0 & 0 \\ 0 & \rho_r^{22} & 0 \\ 0 & 0 & \rho_r^{33} \end{bmatrix}, \rho_r^{ii} = \frac{E_c}{J^i_{r,c}} \left\| \frac{\overrightarrow{J}}{J^i_{r,c}} \right\|^{n_r-1} \tag{3}$$

The coupling of Maxwell's equations and the anisotropic constitutive equations of each sub-domain Ω_r given by the equations (1),(2) and (3) will lead to the following 3D vectorial non-linear diffusion equation :

$$\begin{cases} \dfrac{\partial \beta_{r,i}(u_1,u_2,u_3)}{\partial t} - \dfrac{1}{c_r^i}\triangle u_i = S_{r,i} \ \text{in each} \quad \Omega_r \\ \overrightarrow{\nabla} u_i \cdot \overrightarrow{n} = e_i(t) \ \text{on} \quad \partial\Omega = \partial\Omega_m \end{cases} \tag{4}$$

with $u_1 = E_x/E_c$, $u_2 = E_y/E_c$, $u_3 = E_z/E_c$, $c_r^i = \mu_0 J^i_{r,c}/E_c$, $\beta_{r,i}(u_1,u_2,u_3) = \left(u_1^2+u_2^2+u_3^2\right)^{\frac{1-n_r}{2n_r}} u_i = v_{r,i}$, and $v_{r,1} = J_x/J^1_{r,c}$, $v_{r,2} = J_y/J^2_{r,c}$, $v_{r,3} = J_z/J^3_{r,c}$. the Neumann boundary conditions terms $e_i(t)$ are fluxes built from both Faraday and Gauss law applied outside the heterogeneous media. Moreover, there is no source term in the matrix domain since it is a normal conductor, while there is one in each superconducting sub-domain as expressed as follows:

$$(S_{r,1}, S_{r,2}, S_{r,3}) = \begin{cases} -\overrightarrow{\nabla}\overrightarrow{\nabla}\cdot\left(\overrightarrow{E}/E_c\right) \ \text{in} \quad \Omega_{r\neq m} \\ \overrightarrow{0} \ \text{in} \quad \Omega_{r=m} \end{cases} \tag{5}$$

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III. NUMERICAL RESULTS

A 3-D sample of the media, that consists of two superconducting cylinders embedded in a rectangular cuboid as a resistive matrix, is considered. While the resistive matrix of $n_3 = 1$, length $l_3 = 20.4\mu m$, width $w_3 = 20.4\mu m$ and height $h_3 = 3\mu m$ has, based on the mathematical model proposed above, isotropic parameters $J^1_{3,c} = J^2_{3,c} = J^3_{3,c} = 10^{-3} A/mm^2$, a superconducting cylinder of $n_1 = 50$, radius $r_1 = 7.5\mu m$ and height $h_1 = 1.6\mu m$ is embedded inside another superconducting cylinder of $n_2 = 20$, radius $r_2 = 10\mu m$ and height $h_2 = 2\mu m$, with both having anisotropic critical current density as follows: $J^1_{1,c} = J^1_{2,c} = 2.25A/mm^2$, $J^2_{1,c} = J^2_{2,c} = 4.5A/mm^2$, and $J^3_{1,c} = J^3_{2,c} = 9A/mm^2$.
The uniform critical electric field in the superconducting cylinders is $E_c = 10^{-7}V/mm$. Moreover, the whole media is subjected, on each of its external faces, to the same constant flux whose magnetic induced field vector is collinear with the normal of the boundary surface. Therefore, the diffusion will proceed in all the directions of the space inside the media.
The mesh has 3850 nodes, $\delta t = 0.01\mu s$, and the computation time is $tospecify$ in parallel computing on 32 cores $2Go/core$ RAM. .

The components $J_x/J^r_{1,c}$, $J_y/J^r_{2,c}$ and $J_z/J^r_{3,c}$, with $r = 1, 2, 3$, each have, through Fig.1,2, and 3, an antiperiodic distribution in the studied media due to their boundary conditions.

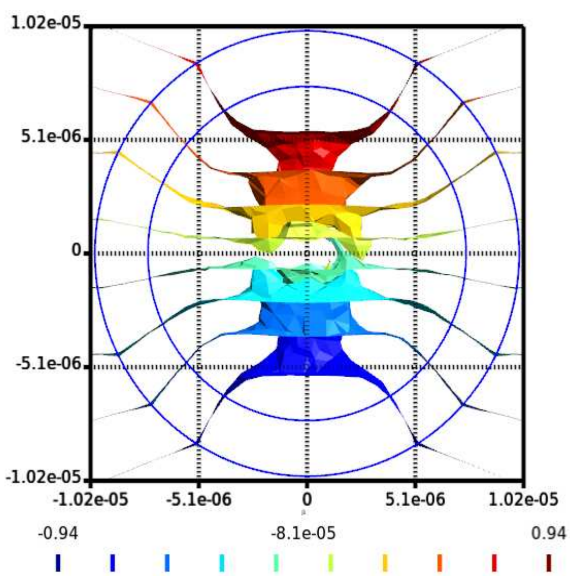

Fig. 1. Partial penetration: $3D$ iso-value distribution of $J_x/J^r_{1,c}$ with $r = 1, 2, 3$ at $t = 10\mu s$

Despite the heterogeneity and anisotropy in the media, all the components of the current density remain continuous in the media. Therefore, the symmetric interior penalty method, used to compute the numerical fluxes at each interface of the mesh, in the discontinuous galerkin method successfully ensures the continuity of the field through each interface.
Moreover, a change in direction of the diffusion process is appearing in each sub-domain of the media. It is certainly due to the change in value of the power exponent n. In this case, as n increases as we move inside the media, we are near the saturation in the superconducting region inside.

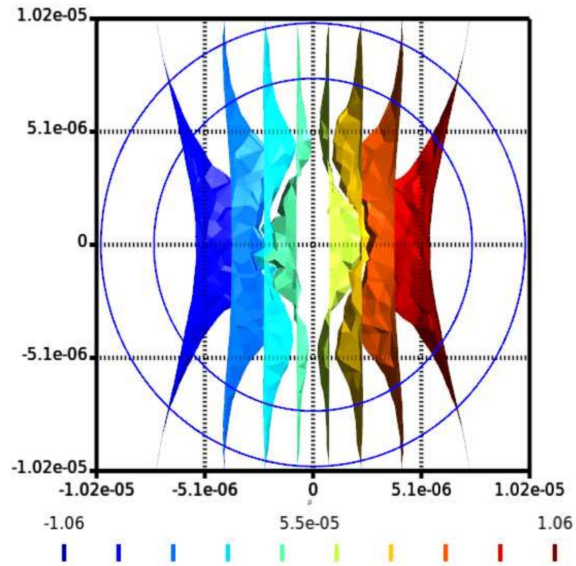

Fig. 2. Partial penetration: $3D$ iso-value distribution of $J_y/J^r_{2,c}$ with $r = 1, 2, 3$ at $t = 10\mu s$

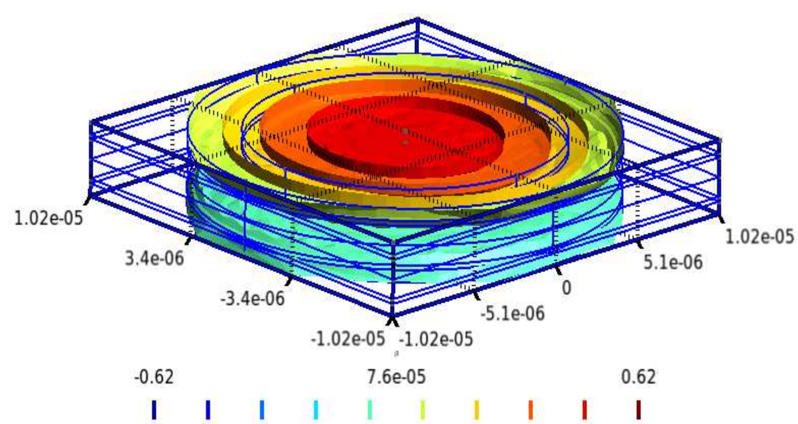

Fig. 3. Partial penetration: $3D$ iso-value distribution of $J_z/J^r_{3,c}$ with $r = 1, 2, 3$ at $t = 10\mu s$

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